The relationship between image noise and spatial resolution of CT scanners

Sue Edyvean, Nicholas Keat, Maria Lewis, Julia Barrett, Salem Sassi, David Platten

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Image noise and spatial resolution

• Aims
  – Describe the origins of the ImPACT Q factor
  – Explore the proportionality relationships
    • in particular; noise against resolution
  – From the findings, look at some implications
The ImPACT Q factor

• Describes a relationship of image quality with respect to dose

\[ Q \propto \sqrt[\frac{f^3}{\sigma^2 z D}} \]

- \( f \) = spatial resolution
- \( \sigma \) = image noise
- \( z \) = slice width
- \( D \) = dose

• High Q factor
  – good ‘image quality’ with low dose
  – high spatial resolution, low noise, narrow slice
The ImPACT Q factor

- Drawn from the proportional relationship

\[ \sigma^2 \propto \frac{f^3}{zD} \]

- \( f = \) spatial resolution expressed as a frequency (c/cm)
- \( \sigma = \) image noise
- \( z = \) slice width
- \( D = \) dose
Theoretical derivation

• Rodney Brookes and Giovanni di-Chiro (1976)
  Statistical limitations in x-ray reconstructive tomography
  Medical Physics Vol 3, No 4 July 1976

• Riederer S.J., Pelc N.J. and Chesler D.A. (1978)
  The Noise Power Spectrum in Computed Tomography
  Physics in Medicine and Biology 1978 23(3), 446-454

\[ \sigma^2(\mu) = \frac{\pi^2 \beta \gamma(E) e^\alpha \mu_{en} E}{1200 \omega^3 z D} \]
Other publications, reports and books

- Bassano D.A. (1980, AAPM Summer School)
  - Specification and Quality Assurance for CT Scanners
  - Measurement of Performance Characteristics of CT Scanners
- Farr RF and Allisy-Roberts PJ (1997)
  - Physics for Medical Imaging
- Seeram E., 2000
  - Computed Tomography, Physical Principles, Clinical Applications and Quality Control (2nd Edition)

\[
\sigma^2 \alpha \frac{f^3}{z^2 D}
\]
Unpacking the relationship

- noise relationship with number of photons
  - established

\[
\sigma^2 \propto \frac{1}{N}
\]

- number of photons
  - proportional to slice thickness and mAs (dose)

\[
\sigma^2 \propto \frac{1}{zD}
\]
Unpacking the relationship

- noise relationship with spatial resolution?

\[
\sigma^2 \propto f^3 \quad \ldots ?
\]

\[
\sigma^2 \propto \frac{f^3}{zD}
\]
Empirical approach

- measure image noise and spatial resolution
- range of convolution kernels

@ constant dose and slice width

\[ \sigma^2 \alpha \frac{f^3}{zD} \]
Image noise

water filled phantom

scan projection
radiograph

image

‘noise’ = standard deviation ($\sigma$) of CT values in region of interest
Spatial resolution

ESF → MTF

high contrast edge

sharp image

smooth image

spatial frequency (cm⁻¹)

MTF (%)

Position Along Edge
Modulation transfer function

• resolution descriptors
  – frequency at $\text{MTF}_{50}$
  – frequency at $\text{MTF}_{10}$
  – Q uses average of $\text{MTF}_{50}$, $\text{MTF}_{10}$

• more recently
  – $\text{MTF}_{80}$
  – $\text{MTF}_{2} \approx \text{‘cut-off’}$
  – integral (area under curve)
Scanners and algorithms used

- IGE Lightspeed
  - h: soft, standard, lung, detail, bone, edge
  - b: soft, standard, lung, detail, bone, edge
- Siemens Volume Zoom
  - h: AH..10,20,30,40,50,60,70
  - b: AB...10,20,30,40,50,60
- Toshiba Aquilion
  - h: FC…20,21,22,23,24,25,26,27,28,30,80
  - b: FC....10,11,12,13,14,30
- Marconi (Philips) MX8000
  - h: A,EB,EC,B,C,D
  - b: A,EC,B,C,D
Siemens VZ, all algorithms, head scans

Average of $MTF_{50}$ and $MTF_{10}$

$\log \% \text{ noise}$ for 40 mGy

$\sigma^2 \propto f^{5.7}$

$R^2 = 0.9916$

Power 5.7

Different algorithms

$\sigma^2 \propto f^3$

Power 3

$\sigma^2 \propto f^{5.7}$

$(f)$ Average of $MTF_{50}$ and $MTF_{10}$
Empirical view of noise versus resolution

• early papers only considered simple algorithms
  – ramp filter and Hanning weighted
  – some filter frequencies boosted very differently
Siemens VZ, low res. algorithms, head scans

Average of MTF_{50} and MTF_{10}

\[ \text{log } \% \text{ noise } (\sigma) \]

Power 4.6

\[ R^2 = 0.9948 \]

Power 3

( f ) Average of MTF_{50} and MTF_{10}
All scanners, all algorithms, body scans

![Graph showing the relationship between log % noise for 40 mGy and average of MTF50 and MTF10.](graph)

- **power 5.1**: $R^2 = 0.999$
- **power 4.8**: $R^2 = 0.9795$
- **power 4.3**: $R^2 = 0.9879$
- **power 5.7**: $R^2 = 0.9511$

Symbols:
- **LightSpeed**
- **Mx8000**
- **Aquilion**
- **VolumeZoom**
All scanners, low res. algs, body scans

\[ (f) \text{ Average of } MTF_{50} \text{ and } MTF_{10} \]

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>R²</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>LightSpeed</td>
<td>0.999</td>
<td>4.9</td>
</tr>
<tr>
<td>Mx8000</td>
<td>0.9795</td>
<td>3.6</td>
</tr>
<tr>
<td>Aquilion</td>
<td>0.9879</td>
<td>3.9</td>
</tr>
<tr>
<td>VolumeZoom</td>
<td>0.9511</td>
<td>3.8</td>
</tr>
</tbody>
</table>
Empirical view of noise versus resolution

• early papers
  – derivations focussed on limiting resolution characteristics
eg detector aperture
### Mean of all scanners, all MTF, all algorithms

**Body scans**

<table>
<thead>
<tr>
<th>parameter</th>
<th>resolution power for sigma $^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
</tr>
<tr>
<td>$mtf_{80}$</td>
<td>3.0</td>
</tr>
<tr>
<td>$mtf_{50}$</td>
<td>4.2</td>
</tr>
<tr>
<td>$mtf_{10}$</td>
<td>5.2</td>
</tr>
<tr>
<td>$mtf_{2}$</td>
<td>3.0</td>
</tr>
<tr>
<td>$mtf_{\text{integral}}$</td>
<td>3.9</td>
</tr>
<tr>
<td>$\text{avg}_{MTF50,10}$</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Empirical view of noise versus resolution

- power factor for $\text{MTF}_{50}$ and $\text{MTF}_{10}$ is greater than 3; ~ between 4 and 5

\[ \sigma^2 \propto \frac{f \approx 4-5}{z \cdot D} \]
Empirical view of noise versus resolution

• how does this affect the calculation of $Q$?
• minimise the influence of the power relationship
  – specify resolution
    • eg. body resolution values of $\text{MTF}_{50} = 3.4$ c/cm, $\text{MTF}_{10} = 6$ c/cm
  – find algorithm giving resolution data closest to that value
  – calculate $Q$

\[
Q \propto \sqrt{\frac{f^3}{\sigma^2 z D}}
\]
All scanners, body scans, lower resolution algorithms

use a specific average $\text{MTF}_{50}$ and $\text{MTF}_{10}$ value

% noise for 40 mGy

Average of $\text{MTF}_{50}$ and $\text{MTF}_{10}$
All scanners, body scans, lower resolution algorithms

\[ \sigma^2 \propto f^3 \]

% noise for 40 mGy

Average of MTF\(_{50}\) and MTF\(_{10}\)
All scanners, body scans, lower resolution algorithms

% noise for 40 mGy

Average of MTF_{50} and MTF_{10}

- LightSpeed
- Mx8000
- Aquilion
- VolumeZoom
Conclusion

• The ImPACT Q factor relies on an established relationship

\[ \sigma^2 \propto \frac{f^3}{zD} \]

• using average resolution parameters from the MTF, the noise squared relationship to resolution is shown to be to a power greater than 3

\[ \sigma^2 \propto \frac{f^{4-5}}{zD} \]

• by choosing algorithms close to a fixed spatial resolution the algorithm dependence is minimised
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Theoretical Relationships

- Rodney Brookes and Giovanni di-Chiro

Statistical error in reconstructed image i.e. image noise

\[ \sigma^2(\mu) = \frac{\pi^2 \beta \gamma(E) e^\alpha \mu_{en} E}{1200 \omega^3 h D} \]

- Logarithmic attenuation
- Energy absorption co-efficient
- Photon energy
- Dose
- Detector aperture
- Slice width
- Average depth dose factor
- Beam spreading factor (non parallel rays)
Theoretical Relationships

\[ \sigma^2(\mu) = \frac{\pi^2 \beta \gamma(E) e^\alpha \mu_{en} E}{1200 \omega^3 z D} \]
Microtomography - Basics

\[ Time \propto \frac{SNR^2}{\rho \mu r^4} \exp\left(\frac{\mu d}{2}\right) \]

- \( SNR \) = signal-to-noise resolution
- \( \rho \) = density
- \( \mu \) = linear attenuation coefficient
- \( r \) = voxel dimension
- \( d \) = object diameter

Courtesy of Dr. E.J. Morton, Department of Physics, University of Surrey
Mean of all scanners, average $\text{MTF}_{50,10}$

<table>
<thead>
<tr>
<th></th>
<th>all algorithms</th>
<th></th>
<th>low res. algorithms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>range +/-</td>
<td>mean</td>
<td>range +/-</td>
</tr>
<tr>
<td>body</td>
<td>5.0</td>
<td>0.7</td>
<td>4.1</td>
<td>0.6</td>
</tr>
<tr>
<td>head</td>
<td>5.5</td>
<td>0.3</td>
<td>4.4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

- $\text{MTF}_{50,10}$ lower by 20% with lower res. algos.
- Head ~ 10% higher than body
- Power factor between 4 and 5
The ImPACT Q factor

- Describes image quality with respect to dose

- High Q (quality) factor
  - good ‘image quality’ at low dose
  - image quality in terms of
    - high spatial resolution, low noise, narrow slice

\[
Q \propto \sqrt{\frac{f^3}{\sigma^2 z D}}
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  - some filter frequencies boosted very differently
  - theory looked at limiting resolution
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